Name _____   **TA's Name** _____

Student Number _____   **Section Number** _____

PHILOSOPHY 2B03   INTRODUCTORY LOGIC

DAY CLASS   **INSTRUCTOR: R. T. W. ARTHUR**

THREE HOURS   **April, 2014**

THIS EXAMINATION PAPER INCLUDES 4 PAGES AND 14 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.
          Calculators are neither needed nor permitted.

> ➢ **SPECIAL INSTRUCTIONS:**
> ➢ **ANSWER ALL 14 QUESTIONS**
> ➢ **Please read the instructions for each question carefully!**
> ➢ **Write your answers in the exam booklet, and hand in this paper with it.**
> ➢ **Write both your name *and your TA section* on your exam booklet.**

1. [**SL proof**]

   (a) *Prove the formal validity* of the following sequent:

   ¬(F & ¬G), ¬F → H, ¬H → ¬G ⊢ H

   [4 marks]

2. [**SL proof**] (a) In the following "proof" of P → ¬O, ¬P → D ⊢ ¬P → (D ∨ O) there are *2 distinct mistakes* made in applying rules of inference (as distinct from any strategic errors). *Identify the mistakes* (explaining them briefly)

(1) P → ¬O	Prem	
(2) ¬P → D	Prem	
(3) ¬P	Supp/CP	
(4) O	1, 3 MT	
(5) D	2, 3 MP	
(6) D ∨ O	5, 4 Disj	
(7) ¬P → (D ∨ O)	3-6 CP	

   (b) *give a correct proof of the formal validity of the sequent in* (b).

   [10 marks]

3. [**SL symbⁿ, proof**] *(a) **Symbolize** the following argument* given by physicist Lee Smolin, *and (b) **give a proof of its validity***:

    If the laws of physics exist ETERNALLY, then they are not rationally COMPREHENSIBLE. But if they are created by NATURAL processes acting in time, then it becomes possible to BRING the laws of physics inside time. And if the laws are brought inside time they will become rationally comprehensible. But the laws of physics either exist eternally or are created by natural processes acting in time, so it follows that they are rationally comprehensible if and only if they can be brought inside time.

    [8 marks]

4. [**SL T-trees**] *Using **the method of truth trees** determine whether each of the following sequents is valid or invalid*:

    (a) (D & O) → ¬I, D ⊢ I → ¬O

    (b) B ↔ (C & D), ¬C ⊢ ¬D ∨ B   [8 marks]

5. [**SL T-table**] ***Determine** using a **truth table** whether the following abstract statement is a **contradiction, a tautology or a contingent statement:***

    [F & (¬G → G)] ↔ (F ∨ ¬G)   [4 marks]

6. [**Carroll diagrams**] ***Determine whether each*** of the arguments in (a) and (b) ***is valid*** or ***invalid*** using the ***Carroll diagram method*** (with a brief explanation of your answer):

    (a) Some NEUTRINOS go FASTER than light. But no neutrinos have rest MASS. It follows that some things with no rest mass go faster than light.

    (b) ARIANS do not BELIEVE in Christ's divinity. Therefore Arians are DEISTS, since no Deists believe in Christ's divinity.

    (c) *Assuming that every predicate is to appear exactly twice in the argument, use a **Carroll diagram** to work out **what conclusion** (express it in colloquial English!) **may be validly inferred** for the following 4-predicate argument*:

    All of the SULTAN'S camels are BACTRIAN. Only DROMEDARIES move FAST. No Bactrian camel is a Dromedary. [UD: camels]

    [10 marks]

Continued on next page…

7. [**PL symbⁿ**] *Symbolize the following statements* of *predicate logic* using the suggested abbreviations:

   (a) There are CALIFORNIANS who do not SURF.

   (b) VEGETARIANS don't eat MEAT.

   (c) <u>Assad</u> is a DICTATOR who FORFEITED the right to rule.

   (d) Every STATEMENT is either TRUE or not PROVEN.

   [4 marks]

8. [**PL symbⁿ + proof**] The following argument is **penevalid**. (a) *Determine the one-predicate existential premise that would render it valid*, (b) *symbolize the argument together with this premise*, and (c) *prove its validity using predicate logic*.

   All MICRO-organisms are too SMALL to see. No FUNGI are LEMONS. But all fungi are micro-organisms. It must then follow that some things that are too small to see are not lemons.

   [10 marks]

9. [**PL proofs**] *Prove the **validity** of the following sequent:*

   Pf, $\forall x(Px \rightarrow Mx) \vdash \exists x (Mx \,\&\, Px)$ [6 marks]

10. [**PL proof**] In the following *incorrect proof of* $\forall x(Fx \rightarrow Gx) \vdash \forall x \neg Gx \rightarrow \exists x \neg Fx$, there are **2 distinct mistakes** *made in applying rules of inference or use of arbitrary names*. (a) ***Identify them*** (explaining them briefly), and

    | (1) $\forall x (Fx \rightarrow Gx)$ | Prem |
    | (2) $\neg Gu$ | Supp/CP |
    | (3) $Fu \rightarrow Gu$ | 1 UI |
    | (4) $\neg Fu$ | 2, 3 MT |
    | (5) $\forall x \neg Gx$ | 2 UG |
    | (6) $\exists x \neg Fx$ | 4 EG |
    | (7) $\forall x \neg Gx \rightarrow \exists x \neg Fx$ | 5-6 CP |

    (b) ***give a correct proof of validity*** of the above sequent.   [10 marks]

11. [**RL symbⁿ**] *Using the suggested notation, **translate** the **relational** statements:*

    (a) <u>Joyce</u> is TALLER₂ than <u>Mario</u>.

    (b) Someone is TALLER₂ than <u>Mario</u>.

(c) All the WOMEN are TALLER$_2$ than <u>Mario</u>.

(d) Some of the WOMEN are TALLER$_2$ than any of the MEN.

[UD: a group of people; xTy := x is taller than y, j := Joyce, m := Mario, Wx := x is a woman, Mx := x is a man]   [4 marks]

12. [**RL proof**] (a) *Prove the validity* of the following sequent in relational logic:

$(\forall x)\neg Rxx$, $(\forall x)(\forall y)(\forall z)\{(Rxy \ \& \ Ryz) \rightarrow Rxz\}$ ⊦ $(\forall x)(\forall y)(Rxy \rightarrow \neg Ryx)$

(b) *What properties* of the relation R *are represented by the premises and conclusion respectively?*   [8 marks]

13. [**wffs**] (a) *Two of the following formulas are **not** wffs. **Identify which two**:*

(i) $\neg(\neg R) \lor S$ (ii) $\exists x\forall y(Ax \rightarrow Dy)$ (iii) $\exists xAx \rightarrow \forall y(Bx \lor Sy)$ (iv) $Am \lor Jr$

(b) *For the two that **are** wffs, **explain briefly** how each of them can be constructed by the rules of wff-formation.*

(c) ***Identify the scope of the existential quantifier in*** (ii) ***and*** (iii).

[6 marks]

14. [**SL, PL**] The following argument is an instance of a fallacy in ***statement logic***:

If some pyramid-shaped hills in Bosnia are ancient pyramids, then Visocica Hill is an ancient pyramid. But it is false that any pyramid-shaped hills in Bosnia are ancient pyramids. Therefore it is false that Visocica Hill is an ancient pyramid.

(a) What ***invalid form*** is this an instance of? What is the fallacy called?

(b) Now treat this argument as ***predicate logic*** argument with the **implicit premise** "Visocica Hill is a pyramid-shaped hill in Bosnia", and ***symbolize*** it with this premise using the dictionary: [UD: pyramid-shaped hills; Bx := x is in Bosnia, Px := x is an ancient pyramid, v := Visocica Hill].

(c) ***Prove the formal validity*** of the resulting argument ***by a predicate logic proof***. (You will find the first premise redundant).

(d) Given the definition of formal validity, is the argument ***formally valid*** or ***invalid***? Explain.

[8 marks]

End of exam